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Higgs-Field Gravity

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It is shown that any excited Higgs field mediates an attractive scalar gravitational interaction of Yukawa type between the elementary particles, which become massive by the ground state of the Higgs field.

1. INTRODUCTION

Until now the origin of the mass of the elementary particles has been unclear. Usually mass is introduced by the interaction with the Higgs field; however, in this way the mass is not explained, but only reduced to the parameters of the Higgs potential, whereby the physical meaning of the Higgs field and its potential remains not understood. We give here a contribution to its interpretation.

There exists an old idea of Einstein (1917), the so-called "principle of relativity of inertia," according to which mass should be produced by the interaction with the gravitational field. Einstein argued that the inertial mass is only a measure for the resistance of a particle against the *relative* acceleration with respect to other particles; therefore, within a consequent theory of relativity the mass of a particle should originate in the interaction with all other particles of the universe (Mach's principle), whereby this interaction should be the gravitational one which couples to all particles, i.e., to their masses or energies. He postulated even that the value of the mass of a particle should go to zero if one puts the particle at an infinite distance from all others.

This fascinating idea was not very successful in Einstein's theory of gravity, i.e., general relativity, although it caused Einstein to introduce the cosmological constant in order to construct a cosmological model with finite space, and Brans and Dicke (1961) to develop their scalar-tensor theory. But an explanation of the mass did not follow from it till now.

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In this paper we show that the successful Higgs-field mechanism lies in the direction of Einstein's idea of producing mass by gravitational interaction. We find that the Higgs field as source of the inertial mass of the elementary particles has something to do with gravity (Dehnen *et al.*, (1990)): it mediates a *scalar gravitational* interaction between *massive* particles however, of Yukawa type, because the Higgs field itself becomes massive after symmetry breaking. On the other hand, an estimation of the coupling constants shows that it is improbable that this Higgs-field gravity can be identified with any experimental phenomenon. Perhaps its applicability lies beyond the scope of present experiments.

2. GRAVITATIONAL FORCE AND POTENTIAL EQUATION

We perform our calculations for the sake of generality with a $U(N)$ model and start from the Lagrange density of fermionic fields coupled with the Higgs field both belonging to the localized group $U(N)$ $[c=1,\ \eta_{\mu\nu}=\operatorname{diag}(1,-1,-1,-1)]$:

$$L=\frac{\hbar}{2}i\bar{\psi}\gamma^{\mu}D_{\mu}\psi+\text{h.c.}-\frac{\hbar}{16\pi}F^{a}{}_{\lambda\mu}F_{a}{}^{\lambda\mu}$$
$$+\tfrac{1}{2}(D_{\mu}\phi)^{+}D^{\mu}\phi-\frac{\mu^{2}}{2}\phi^{+}\phi-\frac{\lambda}{4!}(\phi^{+}\phi)^{2}-k\bar{\psi}\phi^{+}\hat{x}\psi+\text{h.c.} \qquad (1)$$

(μ^2, λ, and k are real parameters of the Higgs potential). Here D_μ represents the covariant derivative with respect to the localized group $U(N)$,

$$D_{\mu}=\partial_{\mu}+igA_{\mu} \qquad (1a)$$

[g is the gauge coupling constant, $A_\mu=A_\mu{}^a\tau_a$ are the gauge potentials, and τ_a are the generators of the group $U(N)$] and the gauge field strength $F_{\mu\nu}$ is determined by its commutator, $F_{\mu\nu}=(1/ig)[D_\mu, D_\nu]=F^a{}_{\mu\nu}\tau_a$; furthermore, $\hat{x}$ is the Yukawa coupling matrix. For applying the Lagrange density (1) to a special model, e.g., the Glashow-Salam-Weinberg model or even the GUT model, the wave function ψ, the generators τ_a, the Higgs field ϕ, and the coupling matrix $\hat{x}$ must be specified explicitly (Dehnen and Frommert, to be published).

From (1) we get immediately the field equations for the spinorial matter fields (ψ fields):

$$i\gamma^{\mu}D_{\mu}\psi-\frac{k}{\hbar}(\phi^{+}\hat{x}+\hat{x}^{+}\phi)\psi=0 \qquad (2)$$

the Higgs field ϕ:

$$D^{\mu}D_{\mu}\phi+\mu^{2}\phi+\frac{\lambda}{3!}(\phi^{+}\phi)\phi=-2k\bar{\psi}\hat{x}\psi \qquad (3)$$

and the gauge fields $F^{a\mu\lambda}$:

$$\partial_\mu F^{a\mu\lambda} + igf^a{}_{bc}A^{b\mu}F^{c\lambda}_\mu = 4\pi j^{a\lambda} \tag{4}$$

with the gauge-current density

$$j^{a\lambda} = g\left(\bar{\psi}\gamma^\lambda \tau^a \psi + \frac{i}{2\hbar}[\phi^+\tau^a D^\lambda \phi - (D^\lambda\phi)^+\tau^a\phi]\right) \tag{4a}$$

Here $f^a{}_{bc}$ are the totally skew-symmetric structure constants of the group $U(N)$. The gauge-invariant canonical energy-momentum tensor reads, with the use of (2),

$$\begin{aligned} T_\lambda{}^\mu = {} & \frac{i\hbar}{2}[\bar{\psi}\gamma^\mu D_\lambda\psi - (D_\lambda\bar{\psi})\gamma^\mu\psi] \\ & - \frac{\hbar}{4\pi}[F^a{}_{\lambda\nu}F_a{}^{\mu\nu} - \tfrac{1}{4}\delta_\lambda{}^\mu F^a{}_{\alpha\beta}F_a{}^{\alpha\beta}] \\ & + \frac{1}{2}\Bigg[(D_\lambda\phi)^+D^\mu\phi + (D^\mu\phi)^+D_\lambda\phi \\ & - \delta_\lambda{}^\mu\left\{(D_\alpha\phi)^+D^\alpha\phi - \mu^2\phi^+\phi - \frac{2\lambda}{4!}(\phi^+\phi)^2\right\}\Bigg] \end{aligned} \tag{5}$$

and satisfies the conservation law

$$\partial_\mu T_\lambda{}^\mu = 0 \tag{6}$$

Obviously, the current density (4a) has a gauge-covariant matter-field and Higgs-field part, i.e., $j^{a\lambda}(\psi)$ and $j^{a\lambda}(\phi)$ respectively, whereas the energy momentum tensor (5) consists of a sum of three gauge-invariant parts:

$$T_\lambda{}^\mu = T_\lambda{}^\mu(\psi) + T_\lambda{}^\mu(F) + T_\lambda{}^\mu(\phi) \tag{7}$$

represented by the brackets on the right-hand side of equation (5).

In view of analyzing the interaction caused by the Higgs field, we investigate first the equation of motion for the expectation value of the 4-momentum of the matter fields and the gauge fields. From (6) and (7) one finds, neglecting surface integrals in the spacelike infinity,

$$\partial_0 \int [T_\lambda{}^0(\psi) + T_\lambda{}^0(F)]\, d^3x = -\int \partial_\mu T_\lambda{}^\mu(\phi)\, d^3x \tag{8}$$

Insertion of $T_\lambda{}^\mu(\phi)$ according to (5) and elimination of the second derivatives of the Higgs field by the field equations (3) results in

$$\frac{\partial}{\partial t}\int [T_\lambda{}^0(\psi)+T_\lambda{}^0(F)]\,d^3x$$
$$=k\int \bar{\psi}[(D_\lambda\phi)^+\hat{x}+\hat{x}^+(D_\lambda\phi)]\psi\,d^3x$$
$$+\frac{ig}{2}\int F^a{}_{\mu\lambda}[\phi^+\tau_a D^\mu\phi-(D^\mu\phi)^+\tau_a\phi]\,d^3x \tag{9}$$

The right-hand side represents the expectation value of the 4-force, which causes the change of the 4-momentum of the ψ fields and the F fields with time. However, the last expression can be rewritten with the use of the field equations (4) as follows:

$$\partial_\mu T_\lambda{}^\mu(F)=\hbar F^a{}_{\mu\lambda}(j_a{}^\mu(\psi)+j_a{}^\mu(\phi)) \tag{9a}$$

One obtains instead of (9)

$$\frac{\partial}{\partial t}\int T_\lambda{}^0(\psi)\,d^3x=\int \hbar F^a{}_{\lambda\mu}j^\mu{}_a(\psi)\,d^3x$$
$$+k\int \bar{\psi}[(D_\lambda\phi)^+\hat{x}+\hat{x}^+(D_\lambda\phi)]\psi\,d^3x \tag{10}$$

where on the right-hand side we have the 4-force of the gauge field and the Higgs field, both acting on the matter field. Evidently, the gauge-field strength couples to the gauge currents, i.e., to the gauge coupling constant g according to (4a), whereas the Higgs-field strength (gradient of the Higgs field) couples to the fermionic mass parameter k (Becher *et al.*, 1981). This fact points to a *gravitational* action of the scalar Higgs field.

2.1. Gravitational Interaction on the Level of the Field Equations

For demonstrating the gravitational interaction explicitly, we perform at first the spontaneous symmetry breaking, because in the case of a scalar gravity only massive particles should interact.[2] For this, $\mu^2<0$ must be valid, and according to (3) and (5), the ground state ϕ_0 of the Higgs field is defined by

$$\phi_0^+\phi_0=v^2=-6\mu^2/\lambda \tag{11}$$

which we resolve as

$$\phi_0=vN \tag{12}$$

[2]The only possible source of a scalar gravity is the trace of the energy-momentum tensor.

with

$$N^+N=1, \qquad \partial_\lambda N \equiv 0 \tag{12a}$$

The general Higgs field ϕ is different from (12) by a local unitary transformation:

$$\phi=\rho UN, \qquad U^+U=1 \tag{13}$$

with

$$\phi^+\phi=\rho^2, \qquad \rho=v+\eta \tag{13a}$$

where η represents the real-valued excited Higgs field.

Now we use the possibility of a unitary gauge transformation which is inverse to (13):

$$\begin{gathered}\phi'=U^{-1}\phi, \qquad \psi'=U^{-1}\psi\\ F'_{\mu\nu}=U^{-1}F_{\mu\nu}U\end{gathered} \tag{14}$$

so that

$$\phi'=\rho N \tag{14a}$$

and perform in the following all calculations in the gauge (14) (unitary gauge). For this we note that in the case of the symmetry breaking of the group G

$$G\to\tilde{G} \tag{15}$$

where $\tilde{G}$ represents the rest-symmetry group, we decompose the unitary transformation:

$$U=\hat{U}\cdot\tilde{U}, \qquad \tilde{U}\in\tilde{G}, \qquad \hat{U}\in G/\tilde{G} \tag{15a}$$

with the isotropy property ($\tau_{\tilde{a}}$ generators of the unbroken symmetry)

$$\tilde{U}N=[\exp(i\lambda^{\tilde{a}}\tau_{\tilde{a}})]N=N \tag{16}$$

so that

$$\tau_{\tilde{a}}N=0 \tag{17}$$

is valid. For $\hat{U}$ we write $\hat{U}=\exp(i\lambda^{\hat{a}}\tau_{\hat{a}})$, where $\tau_{\hat{a}}$ are the generators of the broken symmetry.

Using (12)–(17), the field equations (2)–(4) take the form, avoiding the primes introduced in (14),

$$i\gamma^\mu D_\mu\psi-\frac{\hat{m}}{\hbar}(1+\varphi)\psi=0 \tag{18}$$

$$\partial_\mu F_a^{\ \mu\lambda} + igf_{abc}A^{b\mu}F^c{}_\mu{}^\lambda + \frac{1}{\hbar^2}M^2_{ab}(1+\varphi)^2A^{b\lambda} = 4\pi j^\lambda_a(\psi) \tag{19}$$

$$\partial^\mu\partial_\mu\varphi + \frac{M^2}{\hbar^2}\varphi + \frac{1}{2}\frac{M^2}{\hbar^2}(3\varphi^2+\varphi^3) = -\frac{1}{v^2}\left[\bar\psi\hat m\psi - \frac{1}{4\pi\hbar}M^2_{ab}A^a{}_\lambda A^{b\lambda}(1+\varphi)\right] \tag{20}$$

where $\varphi = \eta/v$ represents the excited Higgs field and

$$\hat m = kv(N^+\hat x + \hat x^+ N) \tag{18a}$$

is the mass matrix of the matter field (ψ field),

$$M^2{}_{ab} = M^2_{\hat a\hat b} = 4\pi\hbar g^2v^2N^+\tau_{(\hat a}\tau_{\hat b)}N \tag{19a}$$

is the symmetric matrix of the mass square of the gauge fields ($A^{\hat a}_\mu$ fields), and

$$M^2 = -2\mu^2\hbar^2, \qquad (\mu^2<0) \tag{20a}$$

is the square of the mass of the Higgs field (φ field). Obviously, in the field equations (18)-(20) the Higgs field φ plays the role of an attractive scalar gravitational potential between the *massive* particles: According to equation (20), the source of φ is the mass of the fermions and of the gauge bosons,[3] whereby this equation linearized with respect to φ is a potential equation of Yukawa type. Accordingly, the potential φ has a finite range

$$l = \hbar/M \tag{21}$$

given by the mass of the Higgs particle and v^{-2} has the meaning of the gravitational constant, so that

$$v^{-2} = 4\pi G\gamma \tag{22}$$

is valid, where G is the Newtonian gravitational constant and γ a dimensionless factor, which compares the strength of the Newtonian gravity with that of the Higgs field and which can be determined only experimentally; see Section 3. On the other hand, the gravitational potential φ acts back on the mass of the fermions and of the gauge bosons according to the field equations (18) and (19). Simultaneously, the equivalence between inertial and passive as well as active gravitational mass is guaranteed. This feature results from the fact that by the symmetry breaking only *one* type of mass is introduced.

[3]The second term in the brackets on the right-hand side of (20) is positive with respect to the signature of the metric.

2.2. Gravitational Interaction on the Level of the Momentum Law

First we consider the potential equation from a more classical standpoint. With respect to the fact of a *scalar* gravitational interaction, we rewrite equation (20) with the help of the trace of the energy-momentum tensor, because this should be the only source of a scalar gravitational potential within a Lorentz-covariant theory. From (5) one finds after symmetry breaking, in analogy to (7),

$$T_\lambda{}^\mu(\psi)=\frac{i\hbar}{2}[\bar{\psi}\gamma^\mu D_\lambda\psi-(D_\lambda\bar{\psi})\gamma^\mu\psi] \tag{23a}$$

$$\begin{aligned}T_\lambda{}^\mu(A)=&-\frac{\hbar}{4\pi}(F^a{}_{\lambda\nu}F_a{}^{\mu\nu}-\tfrac{1}{4}\delta_\lambda{}^\mu F^a{}_{\alpha\beta}F_a{}^{\alpha\beta})\\&+\frac{1}{4\pi\hbar}(1+\varphi)^2M^2{}_{ab}(A^a{}_\lambda A^{b\mu}-\tfrac{1}{2}\delta_\lambda{}^\mu A^a_\nu A^{b\nu})\end{aligned} \tag{23b}$$

$$T_\lambda{}^\mu(\varphi)=v^2\left[\partial_\lambda\varphi\partial^\mu\varphi-\frac{1}{2}\delta_\lambda{}^\mu\left\{\partial_\alpha\varphi\,\partial^\alpha\varphi+\frac{M^2}{4\hbar^2}(1+\varphi)^2(1-2\varphi-\varphi^2)\right\}\right] \tag{23c}$$

From this it follows immediately using the field equation (18) that

$$\begin{aligned}T=T_\lambda{}^\lambda=&\bar{\psi}\hat{m}\psi(1+\varphi)-\frac{1}{4\pi\hbar}M^2{}_{ab}A^a_\lambda A^{b\lambda}(1+\varphi)^2\\&+v^2\left[\frac{M^2}{2\hbar^2}(\varphi^4+4\varphi^3+4\varphi^2-1)-\partial_\lambda\varphi\partial^\lambda\varphi\right]\end{aligned} \tag{23d}$$

The comparison with equation (20) shows that the source of the potential φ is given by the first two terms of (23d), i.e., by $T(\psi)$ and $T(A)$, as expected. In this way we obtain as potential equation using (22)

$$\partial^\mu\partial_\mu\varphi+\frac{M^2}{\hbar^2}\varphi+\frac{1}{2}\frac{M^2}{\hbar^2}(3\varphi^2+\varphi^3)=-4\pi G\gamma(1+\varphi)^{-1}[T(\psi)+T(A)] \tag{24}$$

In the linearized version (with respect to φ) equation (24) represents a potential equation for φ of Yukawa type with the trace of the energy-momentum tensor of the massive fermions and the massive gauge bosons as source.

Finally, we investigate the gravitational force caused by the Higgs field in more detail. Insertion of the symmetry breaking according to (12) up to (17) into the first integral of the right-hand side of (9) yields

$$\begin{aligned}K_\lambda&=k\bar{\psi}[(D_\lambda\phi)^+\hat{x}+\hat{x}^+(D_\lambda\phi)]\psi\\&=\bar{\psi}\hat{m}\psi\partial_\lambda\varphi+v(1+\varphi)[(D_\lambda N)^+k\bar{\psi}\hat{x}\psi+k\bar{\psi}\hat{x}^+\psi D_\lambda N]\end{aligned} \tag{25}$$

Substitution of the conglomerate $k\bar{\psi}\hat{x}\psi$ by the left-hand side of the field equation (3) results, with the use of (13a) and (14a), in

$$K_\lambda = \left[\bar{\psi}\hat{m}\psi - \frac{1}{4\pi\hbar} M^2{}_{ab} A^a_\mu A^{b\mu}(1+\varphi)\right]\partial_\lambda\varphi$$
$$-\frac{1}{4\pi\hbar}\partial_\mu[(1+\varphi)^2 M^2{}_{ab}(A^a{}_\lambda A^{b\mu} - \tfrac{1}{2}\delta_\lambda{}^\mu A^a_\nu A^{b\nu})]$$
$$+\frac{v^2}{2} ig(1+\varphi)^2 F^a{}_{\lambda\mu}[N^+\tau_a D^\mu N - (D^\mu N)^+\tau_a N] \tag{26}$$

By insertion of (26) into the right-hand side of (9), the last term of (26) drops out against the last term of (9), whereas the second term of (26) can be combined with $\partial_\mu T^\mu_\lambda(F)$ to $\partial_\mu T^\mu_\lambda(A)$ according to (23b). In this way we obtain, neglecting surface integrals in the spacelike infinity,

$$\frac{\partial}{\partial t}\int [T_\lambda{}^0(\psi) + T_\lambda{}^0(A)]\, d^3x = \int \left[\bar{\psi}\hat{m}\psi - \frac{1}{4\pi\hbar} M^2{}_{ab} A^a_\mu A^{b\mu}(1+\varphi)\right]\partial_\lambda\varphi\, d^3x \tag{27}$$

In total analogy to the procedure yielding the potential equation (24), we substitute the bracket of the 4-force in (27) by the traces $T(\psi)$ and $T(A)$ given by (23d):

$$\frac{\partial}{\partial t}\int [T_\lambda{}^0(\psi) + T_\lambda{}^0(A)]\, d^3x = \int (1+\varphi)^{-1}[T(\psi) + T(A)]\partial_\lambda\varphi\, d^3x \tag{28}$$

Considering the transition from equation (9) to (10), we can express the time derivative of the 4-momentum of the gauge fields by a 4-force acting on the matter currents. Restricting this procedure to the *massless* gauge fields, we get from (28)

$$\frac{\partial}{\partial t}\int [T_\lambda{}^0(\psi) + T_\lambda{}^0(A_\sigma{}^{\hat{a}})]\, d^3x$$
$$= \int \hbar F^{\hat{a}}{}_{\lambda\mu} j^\mu{}_{\hat{a}}(\psi)\, d^3x + \int (1+\varphi)^{-1}[T(\psi) + T(A_\sigma{}^{\hat{a}})]\partial_\lambda\varphi\, d^3x \tag{29}$$

Here the first term of the right-hand side describes the 4-force of the massless gauge bosons acting on the matter field coupled by the gauge coupling constant g [see (4a)], whereas the second term [identical with the right-hand side of (28)] is the attractive gravitational force of the Higgs field φ acting on the masses of the fermions and of the gauge bosons, which are simultaneously the source of the Higgs potential φ according to (24). This behavior is exactly that of classical gravity, coupling to the mass ($\equiv$ energy) only and not to any charge. However, the qualitative difference with respect to the Newtonian gravity consists, besides the nonlinear terms in (24), in the finite range of φ caused by the Yukawa term.

3. FINAL REMARKS

We want to point to some interesting features of our result. First note that in view of the right-hand side of (28), it is appropriate to define

$$\ln(1+\varphi)=\chi \tag{30}$$

as a new gravitational potential, so that the momentum law reads

$$\frac{\partial}{\partial t}\int [T_\lambda{}^0(\psi)+T_\lambda{}^0(A)]\,d^3x=\int [T(\psi)+T(A)]\partial_\lambda\chi\,d^3x \tag{31}$$

Then the nonlinear terms concerning φ in (24) can be expressed by $T(\varphi)\equiv T(\chi)$ according to the third term of the right-hand side of (23d). In this way the field equation for the potential χ (excited Higgs field) takes the very impressive form

$$\partial_\mu\partial^\mu e^{2\chi}+\frac{M^2}{\hbar^2}e^{2\chi}=-8\pi G\gamma[T(\psi)+T(A)+T(\chi)] \tag{32}$$

Equations (31) and (32) are indeed those of scalar gravity with self-interaction in a natural manner. For the understanding of the Higgs field it may be of interest that the structure of equation (32) exists already before the symmetry breaking. Considering the trace T of the energy-momentum tensor (5), one finds with the use of the field equations (2) and (3)

$$\partial_\mu\partial^\mu(\phi^+\phi)+\frac{M^2}{\hbar^2}(\phi^+\phi)=-2T \tag{33}$$

with $M^2=-2\mu^2\hbar^2$. Accordingly, the Yukawa-like self-interacting scalar gravity of the Higgs field is present within the theory from the very beginning. Equation (33) possesses an interesting behavior with respect to the symmetry breaking. Then from the second term on the left-hand side there results in view of (11) in the first step a cosmological constant $M^2v^2/\hbar^2$; but this is compensated exactly by the trace of the energy-momentum tensor of the ground state. It is our opinion that this is the general property of the cosmological constant, also in general relativity.

Furthermore, because in (21) the mass M is that of the Higgs particle, the range l of the potential φ should be very short, so that until now no experimental evidence for the Higgs gravity exists, at least in the macroscopic limit. For this reason it also appears improbable that it has something to do with the so-called fifth force (Eckhardt *et al.*, 1988). Finally, the factor

γ in (22) can be estimated as follows: Taking into consideration the unified theory of electroweak interactions, the value of v [see (19a)] is correlated with the mass M_W of the W bosons according to $v^{-2} = \pi g_2{}^2 \hbar / M_W^2$ [g_2 is the gauge-coupling constant of the group $SU(2)$]. Combination with (22) results in

$$\gamma = \frac{g_2^2}{2}\left(\frac{M_P}{M_W}\right)^2 = 2 \times 10^{32} \tag{34}$$

(M_P is the Planck mass). Consequently, the Higgs gravity represents a relatively strong scalar gravitational interaction between massive elementary particles, with, however, extremely short range and with the essential property of quantizability. If any Higgs field exists in nature, this gravity is present.

The expression (34) shows that in the case of a symmetry breaking where the bosonic mass is of the order of the Planck mass, the Higgs gravity approaches the Newtonian gravity, if the mass of the Higgs particle is sufficiently small. In this connection the question arises, following Einstein's idea of relativity of inertia, if it is possible to construct a tensorial quantum theory of gravity with the use of the Higgs mechanism, leading at last to Einstein's gravitational theory in the classical macroscopic limit.

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